II. "On Clinant Geometry, as a means of expressing the General Relations of Points in a Plane, realizing Imaginaries, reconciling Ordinary Algebra with Plane Geometry, and extending the Theories of Anharmonic Ratios." By ALEXANDER J. ELLIS, B.A., F.C.P.S. Communicated by ARTHUR CAYLEY, Esq. Received January 28, 1863.

(Abstract.)

The serious difficulties presented by "imaginaries" in plane geometry arise from treating the "principle of signs" as a matter of convention, and not as a particular case of a general operation, here termed a *clinant*, which consists in altering the length of a line in a given ratio, and rotating it through a given angle. As the calculus of clinants furnishes a geometrical representation for every algebraical result, imaginaries disappear, and there is no longer any apparent disagreement between analysis and geometry. Many theories, as, for example, those of anharmonic ratios, hitherto only established for points on a straight line, are also easily extended by means of clinants to embrace any points upon a plane. The object of the present paper is to establish and illustrate these facts. For this purpose it is divided into three distinct but closely connected parts.

Part I. shows that clinants obey the same laws of calculation as ordinary algebraical expressions, and explains their notation and geometrical construction. This is illustrated by the solution of the problem of the determinate section generalized, and by a geometrical explanation of "imaginary" trigonometrical functions, applied to the discovery of the "imaginary" double rays in an homography.

Part II. establishes the theory of *stigmatics*. An *index* point, supposed to move from any *origin* into every point on a plane, is accompanied by one or more satellite points, termed *stigmata*, the relative position of the stigmata and index at any time being dependent on the relative position of the index and origin, according to some assigned law. The locus of the stigmata, corresponding to each *path* of the index, forms a *stigmatic curve*. The aggregate of these curves constitutes a *stigmatic*, which therefore consists of points conjugated with each other according to a *characteristic* law, ulti-

mately expressible by an equation between the clinant of the line connecting the index with the origin and the clinant of the line connecting the stigma and the index. By elimination between two such equations, the common stigmata (*systigmata*) of two stigmatics, and by the condition of equal roots their coalescent systigmata, or *homostigmata*, may be determined. These systigmata and homostigmata include, as particular cases, the points of "real" and "imaginary" intersection and contact of algebraical curves.

These generalities are illustrated by a consideration of the general stigmatic straight line and the central stigmatic circle. The stigmatic straight line consists of stigmatic curves similar to the paths of the index, and their systigmata are the "double points" of similar figures. The stigmata of a stigmatic circle are always harmonically conjugated with the extremities of its axis (with which they always lie either on the same straight line, or the circumference of the same circle), and hence form an "involution" of points on a plane. The construction of the systigmata and homostigmata of a stigmatic straight line, and stigmatic circle, furnishes a complete geometrical explanation and realization of the "imaginary intersections" of straight lines, with "real" and "imaginary" circles, "imaginary tangents" to such circles, and their polars and radical axes and common chords.

Part III. contains an extension of the theories of anharmonic ratios from points on straight lines to any points in a plane, and explains and constructs the homography and involution of such systems of points, with their double points, &c. Constant reference is made throughout this part to M. Chasles's 'Géométrie Supérieure,' to show how his fundamental theories may be interpreted as conclusions in clinant geometry, to explain all cases of "imaginaries," and to establish the fact that "real" and "imaginary" points are only two very particular cases of the general theory of conjugated points.

The whole memoir forms an introduction to a new and practical geometrical calculus, including and interpreting all analytical investigations on plane geometry.